UE Fluids Problem F7 Solution S'08

a) $\Gamma(y) = \Gamma_0 \sqrt{1-\left(\frac{2y}{b}\right)^2}$, $\boxed{\gamma(y) = -\frac{d\Gamma}{dy} = \left(\frac{2}{b}\right)^2 \Gamma_0 \frac{y}{\sqrt{1-(2y/b)^2}}}$

Using $y = \frac{b}{2}\cos\theta$, $\sqrt{1-\left(\frac{2y}{b}\right)^2} = \sqrt{1-\cos^2\theta} = \sin\theta$

$$\boxed{\gamma(\theta) = \frac{2\Gamma_0}{b}\,\frac{\cos\theta}{\sin\theta}}$$

$$I_v = \int_{-b/2}^{b/2} \gamma\, y\, dy = \int_0^{\pi} \gamma \frac{b}{2}\cos\theta \cdot \frac{b}{2}\sin\theta\, d\theta = \int_0^{\pi} \frac{2\Gamma_0}{b}\left(\frac{b}{2}\right)^2 \frac{\cos\theta}{\cancel{\sin\theta}} \cdot \cos\theta\, \cancel{\sin\theta}\, d\theta$$

$$\boxed{I_v = \frac{\Gamma_0 b}{2}\int_0^{\pi} \cos^2\theta\, d\theta = \frac{\pi}{4}\Gamma_0 b}$$

b) $\boxed{I_v = \Gamma_v \frac{b_v}{2} + (-\Gamma_v)\left(-\frac{b_v}{2}\right) = \Gamma_v b_v}$

c) Equating the I_v's : $\boxed{b_v = \frac{\pi}{4} b = 0.785\, b}$

seems to match photo.